# *Calcul des déformations des fils élastiques*

## *Fils élastiques en arc de cercle - Matrice de souplesse*

### *Modèle encastré - libre*

### Définitions

$\psi$ limite d'intégration $\psi = \alpha$ si $\alpha <= \psi_F$ $\psi = \psi_F$ si $\alpha > \psi_F$

$\psi_F$ position angulaire des forces extérieures

$\alpha$ position angulaire du déplacement désiré

$s_{ij}$ déplacement dans la direction *i* due à une force unitaire dans la direction *j*

### Matrice de souplesse

Déplacements unitaires dus à $F_x$

$$s_{11}(\psi_F,\psi,\alpha) := \frac{R^3}{E\cdot I_{33}}\cdot\left[\psi\cdot\sin(\psi_F)\cdot\sin(\alpha) - \left(\sin(\alpha)+\sin(\psi_F)\right)\cdot(1-\cos(\psi)) + \frac{1}{2}\cdot(\psi - \sin(\psi)\cdot\cos(\psi))\right]\cdot\frac{N}{m}$$

$$s_{21}(\psi_F,\psi,\alpha) := \frac{R^3}{E\cdot I_{33}}\cdot\left[\sin(\psi_F)\cdot\sin(\psi) - \frac{1}{2}\cdot\sin(\psi)^2 - \psi\cdot\sin(\psi_F)\cdot\cos(\alpha) + \cos(\alpha)\cdot(1-\cos(\psi))\right]\cdot\frac{N}{m}$$

$s_{31}(\psi_F,\psi,\alpha) := 0$ $s_{41}(\psi_F,\psi,\alpha) := 0$ $s_{51}(\psi_F,\psi,\alpha) := 0$

$$s_{61}(\psi_F,\psi,\alpha) := \frac{R^2}{E\cdot I_{33}}\cdot\left(1-\cos(\psi) - \psi\cdot\sin(\psi_F)\right)\cdot N$$

Déplacements unitaires dus à $F_y$

$$s_{12}(\psi_F,\psi,\alpha) := \frac{R^3}{E\cdot I_{33}}\cdot\left[-\psi\cdot\cos(\psi_F)\cdot\sin(\alpha) + \cos(\psi_F)\cdot(1-\cos(\psi)) + \sin(\psi)\cdot\sin(\alpha) - \frac{1}{2}\cdot\sin(\psi)^2\right]\cdot\frac{N}{m}$$

$$s_{22}(\psi_F,\psi,\alpha) := \frac{R^3}{E\cdot I_{33}}\cdot\left[\psi\cdot\cos(\psi_F)\cdot\cos(\alpha) - \left(\cos(\alpha)+\cos(\psi_F)\right)\cdot\sin(\psi) + \frac{1}{2}\cdot(\psi + \sin(\psi)\cdot\cos(\psi))\right]\cdot\frac{N}{m}$$

$s_{32}(\psi_F,\psi,\alpha) := 0$ $s_{42}(\psi_F,\psi,\alpha) := 0$ $s_{52}(\psi_F,\psi,\alpha) := 0$

$$s_{62}(\psi_F,\psi,\alpha) := \frac{R^2}{E\cdot I_{33}}\cdot\left(-\sin(\psi) + \psi\cdot\cos(\psi_F)\right)\cdot N$$

Déplacements unitaires dus à $F_z$

$s_{13}(\psi_F,\psi,\alpha) := 0$ $s_{23}(\psi_F,\psi,\alpha) := 0$

$$s1_{t33}(\psi_F,\psi,\alpha) := \frac{1}{2}\cdot[(-2+\cos(\alpha)\cdot\cos(\psi) + \sin(\alpha)\cdot\sin(\psi))\cdot\sin(\psi) + \cos(\alpha)\cdot\psi]\cdot\cos(\psi_F)$$

$$s2_{t33}(\psi_F,\psi,\alpha) := \frac{1}{2}\cdot[(2-\sin(\alpha)\cdot\sin(\psi) - \cos(\alpha)\cdot\cos(\psi))\cdot\cos(\psi) + \sin(\alpha)\cdot\psi - 2 + \cos(\alpha)]\cdot\sin(\psi_F)$$

$$s3_{t33}(\psi_F,\psi,\alpha) := (\psi - \cos(\alpha)\cdot\sin(\psi) + \sin(\alpha)\cdot\cos(\psi) - \sin(\alpha))$$

$$s_{t33}(\psi_F,\psi,\alpha) := \frac{R^3}{G\cdot J_t}\cdot\left(s1_{t33}(\psi_F,\psi,\alpha) + s2_{t33}(\psi_F,\psi,\alpha) + s3_{t33}(\psi_F,\psi,\alpha)\right)\cdot\frac{N}{m}$$

$$s1_{f33}(\psi_F,\psi,\alpha) := \frac{1}{2}\cdot[(\sin(\alpha)\cdot\sin(\psi)+\cos(\alpha)\cdot\cos(\psi))\cdot\cos(\psi)+\sin(\alpha)\cdot\psi-\cos(\alpha)]\cdot\sin(\psi_F)$$

$$s2_{f33}(\psi_F,\psi,\alpha) := \frac{1}{2}\cdot[\cos(\alpha)\cdot\psi+(\sin(\alpha)\cdot\cos(\psi)-\cos(\alpha)\cdot\sin(\psi))\cdot\cos(\psi)-\sin(\alpha)]\cdot\cos(\psi_F)$$

$$s_{f33}(\psi_F,\psi,\alpha) := \frac{R^3}{E\cdot I_{22}}\cdot(s1_{f33}(\psi_F,\psi,\alpha)+s2_{f33}(\psi_F,\psi,\alpha))\cdot\frac{N}{m}$$

$$s_{33}(\psi_F,\psi,\alpha) := s_{t33}(\psi_F,\psi,\alpha)+s_{f33}(\psi_F,\psi,\alpha)$$

$$s_{t43}(\psi_F,\psi,\alpha) := \frac{R^2}{2\cdot(G\cdot J_t)}\cdot\left[\sin(\psi)^2\cdot\cos(\psi_F)+(-\cos(\psi)\cdot\sin(\psi)+\psi)\cdot\sin(\psi_F)-2+2\cdot\cos(\psi)\right]\cdot N$$

$$s_{f43}(\psi_F,\psi,\alpha) := \frac{R^2}{2\cdot(E\cdot I_{22})}\cdot\left[(\psi+\cos(\psi)\cdot\sin(\psi))\cdot\sin(\psi_F)-\sin(\psi)^2\cdot\cos(\psi_F)\right]\cdot N$$

$$s_{43}(\psi_F,\psi,\alpha) := s_{t43}(\psi_F,\psi,\alpha)+s_{f43}(\psi_F,\psi,\alpha)$$

$$s_{t53}(\psi_F,\psi,\alpha) := \frac{R^2}{2\cdot(G\cdot J_t)}\cdot\left[(-\cos(\psi)\cdot\sin(\psi)-\psi)\cdot\cos(\psi_F)-\sin(\psi)^2\cdot\sin(\psi_F)+2\cdot\sin(\psi)\right]\cdot N$$

$$s_{f53}(\psi_F,\psi,\alpha) := \frac{R^2}{2\cdot(E\cdot I_{22})}\cdot\left[(\cos(\psi)\cdot\sin(\psi)-\psi)\cdot\cos(\psi_F)+\sin(\psi)^2\cdot\sin(\psi_F)\right]\cdot N$$

$$s_{53}(\psi_F,\psi,\alpha) := s_{t53}(\psi_F,\psi,\alpha)+s_{f53}(\psi_F,\psi,\alpha) \qquad s_{63}(\psi_F,\psi,\alpha) := 0$$

Déplacements unitaires dus à $C_x$

$$s_{14}(\psi_F,\psi,\alpha) := 0 \qquad s_{24}(\psi_F,\psi,\alpha) := 0$$

$$s_{t34}(\psi_F,\psi,\alpha) := \frac{R^2}{2\cdot(G\cdot J_t)}\cdot\left[-\cos(\alpha)\cdot\cos(\psi)^2+(2-\sin(\alpha)\cdot\sin(\psi))\cdot\cos(\psi)+\cos(\alpha)+\sin(\alpha)\cdot\psi-2\right]\cdot N$$

$$s_{f34}(\psi_F,\psi,\alpha) := \frac{R^2}{2\cdot(E\cdot I_{22})}\cdot\left(-\cos(\alpha)\cdot\sin(\psi)^2+\sin(\alpha)\cdot\cos(\psi)\cdot\sin(\psi)+\sin(\alpha)\cdot\psi\right)\cdot N$$

$$s_{34}(\psi_F,\psi,\alpha) := s_{t34}(\psi_F,\psi,\alpha)+s_{f34}(\psi_F,\psi,\alpha)$$

$$s_{44}(\psi_F,\psi,\alpha) := \left[\frac{R}{2\cdot(G\cdot J_t)}\cdot(-\cos(\psi)\cdot\sin(\psi)+\psi)+\frac{R}{2\cdot(E\cdot I_{22})}\cdot(\cos(\psi)\cdot\sin(\psi)+\psi)\right]\cdot N\cdot m$$

$$s_{54}(\psi_F,\psi,\alpha) := \left[\frac{-R}{2\cdot(G\cdot J_t)}\cdot\sin(\psi)^2+\frac{R}{2\cdot(E\cdot I_{22})}\cdot\sin(\psi)^2\right]\cdot N\cdot m \qquad s_{64}(\psi_F,\psi,\alpha) := 0$$

Déplacements unitaires dus à $C_y$

$$s_{15}(\psi_F,\psi,\alpha) := 0 \qquad s_{25}(\psi_F,\psi,\alpha) := 0$$

$$s_{t35}(\psi_F,\psi,\alpha) := \frac{R^2}{2\cdot(G\cdot J_t)}\cdot\left(\sin(\alpha)\cdot\cos(\psi)^2-\cos(\alpha)\cdot\cos(\psi)\cdot\sin(\psi)-\cos(\alpha)\cdot\psi+2\cdot\sin(\psi)-\sin(\alpha)\right)\cdot N$$

$$s_{f35}(\psi_F,\psi,\alpha) := \frac{R^2}{2\cdot(E\cdot I_{22})}\cdot\left(-\sin(\alpha)\cdot\cos(\psi)^2+\cos(\alpha)\cdot\cos(\psi)\cdot\sin(\psi)-\cos(\alpha)\cdot\psi+\sin(\alpha)\right)\cdot N$$

$$s_{35}(\psi_F,\psi,\alpha) := s_{t35}(\psi_F,\psi,\alpha)+s_{f35}(\psi_F,\psi,\alpha)$$

$$s_{45}(\psi_F,\psi,\alpha) := \left[\frac{-R}{2\cdot(G\cdot J_t)}\cdot \sin(\psi)^2 + \frac{R}{2\cdot(E\cdot I_{22})}\cdot \sin(\psi)^2\right]\cdot N\cdot m$$

$$s_{55}(\psi_F,\psi,\alpha) := \left[\frac{R}{2\cdot(G\cdot J_t)}\cdot(\cos(\psi)\cdot\sin(\psi)+\psi) + \frac{R}{2\cdot(E\cdot I_{22})}\cdot(-\cos(\psi)\cdot\sin(\psi)+\psi)\right]\cdot N\cdot m$$

$$s_{65}(\psi_F,\psi,\alpha) := 0$$

Déplacements unitaires dus à $C_z$

$$s_{16}(\psi_F,\psi,\alpha) := \frac{R^2}{E\cdot I_{33}}\cdot[-\psi\cdot\sin(\alpha)+(1-\cos(\psi))]\cdot N$$

$$s_{26}(\psi_F,\psi,\alpha) := \frac{R^2}{E\cdot I_{33}}\cdot(-\sin(\psi)+\psi\cdot\cos(\alpha))\cdot N$$

$$s_{36}(\psi_F,\psi,\alpha) := 0 \qquad s_{46}(\psi_F,\psi,\alpha) := 0 \qquad s_{56}(\psi_F,\psi,\alpha) := 0$$

$$s_{66}(\psi_F,\psi,\alpha) := \frac{R}{E\cdot I_{33}}\cdot(\psi)\cdot N\cdot m$$

**Matrice de souplesse**

$$\mathbf{S}(\psi_F,\psi,\alpha) := \begin{pmatrix} s_{11}(\psi_F,\psi,\alpha) & s_{12}(\psi_F,\psi,\alpha) & s_{13}(\psi_F,\psi,\alpha) & s_{14}(\psi_F,\psi,\alpha) & s_{15}(\psi_F,\psi,\alpha) & s_{16}(\psi_F,\psi,\alpha) \\ s_{21}(\psi_F,\psi,\alpha) & s_{22}(\psi_F,\psi,\alpha) & s_{23}(\psi_F,\psi,\alpha) & s_{24}(\psi_F,\psi,\alpha) & s_{25}(\psi_F,\psi,\alpha) & s_{26}(\psi_F,\psi,\alpha) \\ s_{31}(\psi_F,\psi,\alpha) & s_{32}(\psi_F,\psi,\alpha) & s_{33}(\psi_F,\psi,\alpha) & s_{34}(\psi_F,\psi,\alpha) & s_{35}(\psi_F,\psi,\alpha) & s_{36}(\psi_F,\psi,\alpha) \\ s_{41}(\psi_F,\psi,\alpha) & s_{42}(\psi_F,\psi,\alpha) & s_{43}(\psi_F,\psi,\alpha) & s_{44}(\psi_F,\psi,\alpha) & s_{45}(\psi_F,\psi,\alpha) & s_{46}(\psi_F,\psi,\alpha) \\ s_{51}(\psi_F,\psi,\alpha) & s_{52}(\psi_F,\psi,\alpha) & s_{53}(\psi_F,\psi,\alpha) & s_{54}(\psi_F,\psi,\alpha) & s_{55}(\psi_F,\psi,\alpha) & s_{56}(\psi_F,\psi,\alpha) \\ s_{61}(\psi_F,\psi,\alpha) & s_{62}(\psi_F,\psi,\alpha) & s_{63}(\psi_F,\psi,\alpha) & s_{64}(\psi_F,\psi,\alpha) & s_{65}(\psi_F,\psi,\alpha) & s_{66}(\psi_F,\psi,\alpha) \end{pmatrix}$$

$$\mathbf{S_F}(\psi_F,\alpha) := \mathbf{S}(\psi_F,\alpha,\alpha)\cdot(\alpha\le\psi_F) + \mathbf{S}(\psi_F,\psi_F,\alpha)\cdot(\alpha>\psi_F)$$

## *Cas particulier d'une déformation plane*

$$\mathbf{S_P}(\psi_F,\psi,\alpha) := \begin{pmatrix} s_{11}(\psi_F,\psi,\alpha) & s_{12}(\psi_F,\psi,\alpha) & s_{16}(\psi_F,\psi,\alpha) \\ s_{21}(\psi_F,\psi,\alpha) & s_{22}(\psi_F,\psi,\alpha) & s_{26}(\psi_F,\psi,\alpha) \\ s_{61}(\psi_F,\psi,\alpha) & s_{62}(\psi_F,\psi,\alpha) & s_{66}(\psi_F,\psi,\alpha) \end{pmatrix}$$

$$\mathbf{S_{PF}}(\psi_F,\alpha) := \mathbf{S_P}(\psi_F,\alpha,\alpha)\cdot(\alpha\le\psi_F) + \mathbf{S_P}(\psi_F,\psi_F,\alpha)\cdot(\alpha>\psi_F)$$